

# A New Damage Location Parameter for Beam Structures Based on Mode Shape Slope

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**Abstract**— A new damage parameter, called mode shape slope damage factor (MSSDF), is proposed to predict the damage location in a beam structure. In this paper MSSDF utilized the mode shape data yielded by finite element models of intact and damaged beams. The mode shape slope is evaluated using the first derivative approximation formulas at the leftmost and at the rightmost point. Numerical results show that the proposed damage index successfully predicts damage location in the single and multiple damage cases. The MSSDF is more sensitive to damage when compared with other well-known damage index in the literature.

**Index Terms**— beam structure, crack, damage detection, damage index, finite element model, mode shape curvature, mode shape slope.

## 1 INTRODUCTION

CRACKS are one of the main causes of structural failure. In order to reduce or eliminate the sudden failure of structures, they should be regularly checked for cracks and other damages. In recent years vibration based damage detection techniques have been extensively researched. Detailed reviews of the damage indices based on modal parameters have been proposed by Doebling et al. [1], [2], Sohn et al. [3], Guan and Karbhari [4], Sinou [5]. The basic assumption of vibration based damage detection methods is that the changes in structural parameters (stiffness, mass, flexibility) will induce reduction in the modal parameters (natural frequencies and mode shapes). They have certain advantage compared to other nondestructive methods like ultrasonic testing, lamb wave, X-ray, acoustic emission, etc. Vibration based damage detection methods could be applied to inaccessible components and could assess the severity of damages of the entire structure. They are relatively cheap and quick.

According Rytter [6] structural health monitoring could be divided to four different stages. The first stage is the determination of the presence of the damage in the structure. The next stage is the localization of the damage. The third stage is the assessment of the damage severity. The final stage is the prediction of the remaining life of the structure. It uses the information from the previous three stages and is associated with the field of fatigue analysis, structural design assessment and fracture mechanics.

There are different classifications of vibration based damage identification methods using various criteria [1], [2], [3], [4], [5], [7]. A significant part of these methods is based on natural frequency changes [8], [9], [10], [11]. Salawu [12] proposed a detailed review of them. The natural frequency shifts as damage indicators are successfully applied to small simple laboratory structures with a single crack. Unlike the natural frequency, which is a global characteristic of the structure, the mode shape contains spatial information about the structure. Many authors proposed damage detection methods based on changes in mode shape. Two commonly used methods are

modal assurance criterion (MAC) [13] and coordinate modal assurance criterion (COMAC) [14]. The MAC and COMAC values are obtained by comparing two sets of mode shapes (e.g. mode shapes in the undamaged and in the damaged state). Another method, which utilizes the mode shape changes, is structural translational and rotational error checking (STRECH) [15]. Using error localization techniques STRECH locates stiffness differences between two modal models. As an alternative to mode shape based methods are the methods based on mode shape derivatives. Pandey et al [16] first used mode shape curvature (MSC) for damage indicator. They found that the modal curvature is more sensitive to damage than COMAC. Ho and Ewins [17] proposed other damage indicators based on mode shape (mode shape amplitude comparison, flexibility index) and its derivative (mode shape slope, mode shape curvature square). Stubbs et al [18] presented a method based on the decrease in modal strain energy between two structural degrees of freedom, which is defined by the mode shape curvature. The main drawback of the mode shape based methods is the necessity of having measurements from a relatively large number of locations. Pandey and Biswas [19] presented a method based on change in measured flexibility of the structure. Unity check method, proposed by Lim [20], is based on the pseudoinverse relationship between the dynamically measured flexibility matrix and structural stiffness matrix. The flexibility matrix relates the applied loads and resulting structural displacements. It is defined as the inverse of the stiffness matrix. The damage detection methods based on dynamically measured flexibility are not sensitive to small damage. Other vibration-based damage detection methods utilize frequency response function (FRF) [21], [22]. The FRF data contain much more information than the modal data, which covers the range around the resonances. The main disadvantage of the FRF methods is that the accuracy of damage detection strongly depends on the number and location of measurement points. There are damage identification methods based on updating physical parameters of the numerical model (in most cases finite element model) of the structure to match as closely as possible the measured static or dynamic response data [23], [24]. Some of these model updating methods can be used to assess of the damage severity. The drawback of the model updating methods is the necessity of creat-

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ing numerical model. They aren't applicable to small scale damages on a physical model.

In this paper, a new damage detection parameter based on the change of mode shape slope is proposed. The principle of the index is based on comparison of differences of mode shape slopes, which are evaluated using the first derivative approximation formulas at the leftmost and at the rightmost point, obtained from mode shape displacements of intact and damaged beam. To assess the efficiency and the accuracy of the proposed damage location parameter one test example with single and multiple damages is considered. The results obtained are compared with the results obtained by the commonly used crack location method.

## 2 DAMAGE DETECTION METHODS BASED ON MODE SHAPE DATA

Presence of damage changes the mode shape and its derivatives as the deviations are higher near the damaged cross section. These changes could be used to determine the location of single and multiple damages. In this section the most common damage detection methods based on mode shape data are presented.

### 2.1 Methods based on direct change in mode shapes

The methods based on direct change in mode shapes compare two set of series of mode shape (A and B). The sets could be analytical, experimental or analytical and experimental [25]. The index of the modal assurance criterion (MAC) [13] is defined as:

$$MAC_{(i,j)} = \frac{\left( \sum_{k=1}^n \Phi_{i,k}^A \cdot \Phi_{j,k}^B \right)^2}{\sum_{k=1}^n \left( \Phi_{i,k}^A \right)^2 \cdot \sum_{k=1}^n \left( \Phi_{j,k}^B \right)^2}; \quad i = 1, \dots, N_A; \quad j = 1, \dots, N_B \quad (1)$$

where  $k=1, \dots, n$  is the number of considered coordinate of the mode shape,  $\Phi_{i,k}^A$  is the  $k^{\text{th}}$  coordinate of  $i^{\text{th}}$  mode shape of state A,  $\Phi_{j,k}^B$  is the  $k^{\text{th}}$  coordinate of  $j^{\text{th}}$  mode shape of state B,  $N_A$  and  $N_B$  are the number of investigated modes for the state A and B, respectively.

The MAC value varies between 0 and 1. A value of 1 indicates a perfect correlation between the two mode shapes. A value of 0 means that there is a mismatch between the two mode shapes. The MAC criterion doesn't give information about the location of places with differences in considered mode shapes. It only indicates presence of damages in the structure.

The coordinate modal assurance criterion (COMAC) is based on MAC, unlike MAC, it gives information about the damage location. The COMAC takes into account the deviation between two sets of mode shapes for each considered coordinate [14]:

$$COMAC_{(k)} = \frac{\left( \sum_{i=1}^N \Phi_{i,k}^A \cdot \Phi_{i,k}^B \right)^2}{\sum_{i=1}^N \left( \Phi_{i,k}^A \right)^2 \cdot \sum_{i=1}^N \left( \Phi_{i,k}^B \right)^2} \quad (2)$$

where  $N$  is the number of correlated mode shapes of the two sets,  $\Phi_{i,k}^A$  and  $\Phi_{j,k}^B$  are the  $k^{\text{th}}$  coordinate of  $i^{\text{th}}$  mode shape of

state A and B, respectively. The COMAC value is 1 when at coordinate  $k$  the modal displacements from the two set are identical. The COMAC value tending to 0 signifies the possible location of damage.

### 2.2 Mode shape curvature method

The mode shape derivatives are commonly used to locate the damage in the structure. The curvature of a beam is inversely proportional to the flexural stiffness  $EI$  of the beam. The damage present at a certain cross section of the beam reduces the stiffness  $EI$  which increases the curvature. The change in curvature is local and indicates the damage location. Pandey et al [16] use the absolute difference between mode shape curvature of damaged and undamaged beam. They named the proposed indicator mode shape curvature (MSC):

$$MSC_{(k)} = \sum_{i=1}^N \left| \left( \Phi_{i,k}^d \right)'' - \left( \Phi_{i,k}^u \right)'' \right| \quad (3)$$

where  $N$  is the number of considered mode shapes,  $\left( \Phi_{i,k}^d \right)''$  and  $\left( \Phi_{i,k}^u \right)''$  are the mode shape curvature at  $k^{\text{th}}$  coordinate of  $i^{\text{th}}$  mode shape of damaged and undamaged beam, respectively.

Pandey et al compute the mode shape curvature  $\left( \Phi_{i,k} \right)''$  at  $k^{\text{th}}$  coordinate of  $i^{\text{th}}$  mode shape using central difference approximation for second derivate:

$$\left( \Phi_{i,k} \right)'' = \frac{\Phi_{i,k-1} - 2\Phi_{i,k} + \Phi_{i,k+1}}{h_m^2} \quad (4)$$

where  $\Phi_{i,k}$  is the modal displacement for the  $i^{\text{th}}$  mode at  $k^{\text{th}}$  coordinate,  $h_m$  is the distance between measurement coordinates.

Abdel Wahab and De Roeck [26] proposed new index named curvature damage factor (CDF). It averages the absolute difference between mode shape curvature of damaged and undamaged beam for considered mode shapes:

$$CDF_{(k)} = \frac{1}{N} \sum_{i=1}^N \left| \left( \Phi_{i,k}^d \right)'' - \left( \Phi_{i,k}^u \right)'' \right| \quad (5)$$

The large positive peak values of MSC and CDF indicate the possible damage locations.

## 3 THE PROPOSED DAMAGE LOCATION PARAMETER

In this study, a new damage location parameter based on the mode shape slope changes is developed. The mode shape slope is computed through the first derivative approximation formulas at the leftmost and at the rightmost point from the mode shape data. The damage index compares the differences between the results obtained using the two approximation formulas of intact and damaged beam structure.

The mode shapes of an undamaged beam are smooth curves. The beam mode shape has a kink at cross section with a crack or damage. Therefore, at this place the mode shape slope from the left is different from those from the right.

The mode shape slope at any given point along the length of the beam is the first derivative of mode shape at the same location. There are different first derivative approximation formulas [27] such as three-point midpoint formula, three-point endpoint formula, five-point midpoint formula etc. The endpoint formulas are two types: one calculates the first deri-

vate at the leftmost point of the considered mode shape displacement coordinates:

$$(\Phi_{i,k})'_l = \frac{-3\Phi_{i,k} + 4\Phi_{i,k+1} - \Phi_{i,k+2}}{2h_m}; \quad (6)$$

and the other calculates the first derivate at the rightmost point:

$$(\Phi_{i,k})'_r = \frac{\Phi_{i,k-2} - 4\Phi_{i,k-1} + 3\Phi_{i,k}}{2h_m}, \quad (7)$$

where  $\Phi_{i,k}$  is the modal displacement for the  $i^{\text{th}}$  mode at  $k^{\text{th}}$  coordinate,  $h_m$  is the distance between measurement coordinates,  $(\Phi_{i,k})'_l$  and  $(\Phi_{i,k})'_r$  are mode shape slopes at  $k^{\text{th}}$  coordinate of  $i^{\text{th}}$  mode shape using three-point endpoint approximation formulas for first derivate at the leftmost and at the rightmost point, respectively.

The difference between results calculated using (6) and using (7) at  $k^{\text{th}}$  coordinate of  $i^{\text{th}}$  mode shape  $\Delta(\Phi_{i,k})'$  is:

$$\begin{aligned} \Delta(\Phi_{i,k})' &= (\Phi_{i,k})'_r - (\Phi_{i,k})'_l = \\ &= \frac{\Phi_{i,k-2} - 4\Phi_{i,k-1} + 6\Phi_{i,k} - 4\Phi_{i,k+1} + \Phi_{i,k+2}}{2h_m}. \end{aligned} \quad (8)$$

The difference  $\Delta(\Phi_{i,k})'$  increases at peaks and valleys of the mode shape, but the larger values of difference are obtained at the mode shape kinks. The variation of the differences between the results calculated with the two endpoint formulas of damaged beam vs. undamaged beam is the proposed new damage parameter. It is named mode shape slope damage factor (MSSDF) and is expressed by the following relation:

$$MSSDF_{(k)} = \frac{|\Delta(\Phi_k^d)' - \Delta(\Phi_k^u)'|}{\max |\Delta(\Phi_{k=1,\dots,n}^d)' - \Delta(\Phi_{k=1,\dots,n}^u)'|}, \quad (9)$$

where  $\Delta(\Phi_k^d)'$  and  $\Delta(\Phi_k^u)'$  are the difference between mode shape slopes calculated using (8) at  $k^{\text{th}}$  coordinate of considered mode shape of damaged and undamaged beam, respectively.

The points with large MSSDF indicate the possible locations of damage.

#### 4 NUMERICAL EXAMPLES

In order to demonstrate the capabilities of the proposed damage detection parameter a test example is considered. The numerical analysis is carried out for a simply supported beam of square cross section (Fig. 1). The dimensions of the beam are: length,  $L=3$  m, height,  $h=0.1$  m, width,  $b=0.1$  m. The material properties are: modulus of elasticity,  $E=2.1 \times 10^{11}$  N/m<sup>2</sup>, density,  $\rho=7850$  kg/m<sup>3</sup>, Poisson's coefficient,  $\nu=0.3$ .

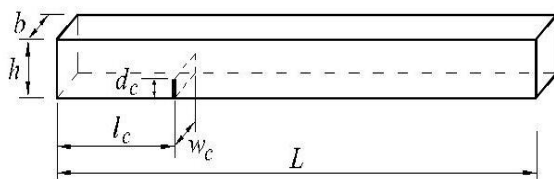


Fig. 1. Beam and crack geometry

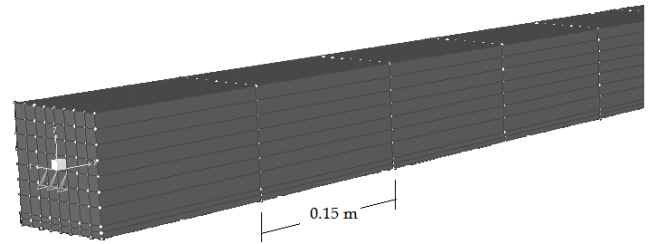


Fig. 2. Finite element model of the beam

The mode shape displacements for the different cases are calculated by the finite element method, using a computer program (SAP 2000). The beam is discretized by 8-node solid elements, which divide the cross section into 80 parts, as can be seen in Fig. 2. All solid elements have equal length (0.15 m). The cracks are modelled by disconnecting joints of certain solid elements. The main crack parameters are crack location  $l_c$ , depth  $d_c$  and width  $w_c$ , as shown in Fig. 2. The damage indices are calculated using MATLAB [28].

Fifteen damage cases are considered. The parameters of cracks for different cases are given in Table 1.

TABLE 1  
PARAMETERS OF CRACKS FOR DIFFERENT DAMAGE CASES

Case number	Crack location $l_c$ [m]	Crack depth $d_c$ [m]	Crack width $w_c$ [m]
1	0.3	0.010	0.100
2	0.6	0.010	0.100
3	0.9	0.010	0.100
4	1.2	0.010	0.100
5	1.5	0.010	0.100
6	1.2	0.005	0.100
7	1.2	0.002	0.100
8	1.2	0.002	0.075
9	1.2	0.002	0.050
10	1.2	0.002	0.025
11	0.3	0.010	0.100
	0.6	0.010	0.100
12	0.3	0.010	0.100
	1.2	0.010	0.100
13	0.3	0.010	0.100
	1.8	0.010	0.100
14	0.3	0.010	0.100
	2.4	0.010	0.100
15	0.3	0.010	0.100
	1.2	0.010	0.100
	2.4	0.010	0.100

#### 4.1 Plots of MSSDF for the first three modes

Plots of MSSDF vs. considered cross section position along the length of the beam for the three mode shapes corresponding to the damage case 3 and case 5 are presented in Fig. 3 and Fig. 4, respectively. As can be observed in the figures, the damage could be identified from the graphs for the first modes. In Fig. 3a and Fig. 4a the damage index at cross section with crack has value equal to 1 and for the other grid points the value of MSSDF is less than 0.1. For the second and third modes corresponding to the damage case 3 (Fig. 3b and Fig. 3c) and for the third mode of case 5 (Fig. 4c) the MSSDF is equal to 1 at damage location, but the points next to it have value more than 0.5.

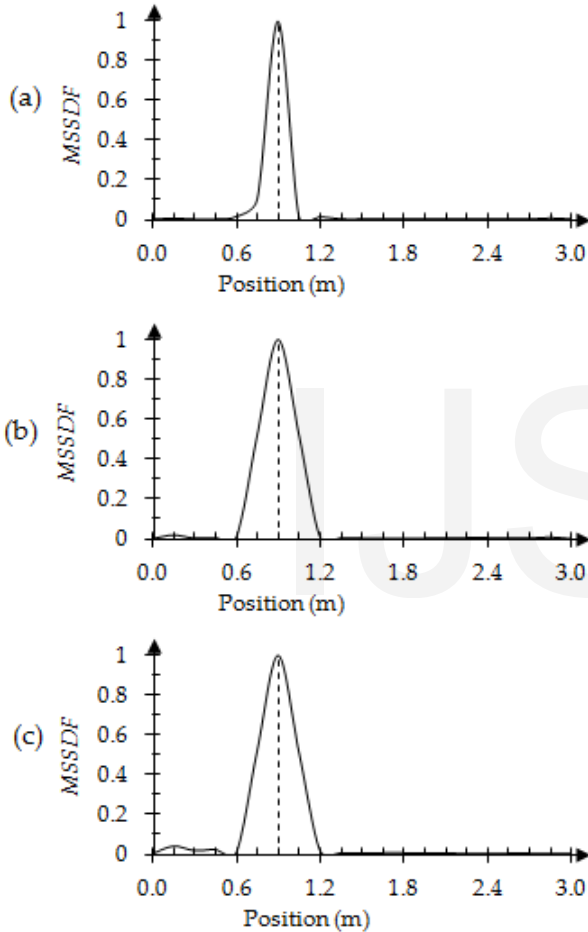


Fig. 3. Plots of damage parameter MSSDF vs. considered cross section position along the beam corresponding to the damage case 3 for: (a) the first mode shape, (b) the second mode shape, (c) the third mode shape

The graph of MSSDF for the second mode corresponding to the damage case 5 (Fig. 4b) differs from the other MSSDF plots. The crack location coincides with the point of asymmetry of mode shape displacements. At those points according to (8) MSSDF is equal to 0. In consequence of presented facts only the first mode shape of the beam is considered.

#### 4.2 Damage Location According MSSDF and nCDF

To assess the efficiency and the accuracy of the proposed damage location parameter the results of MSSDF given by (9) are compared with the results of the commonly used crack

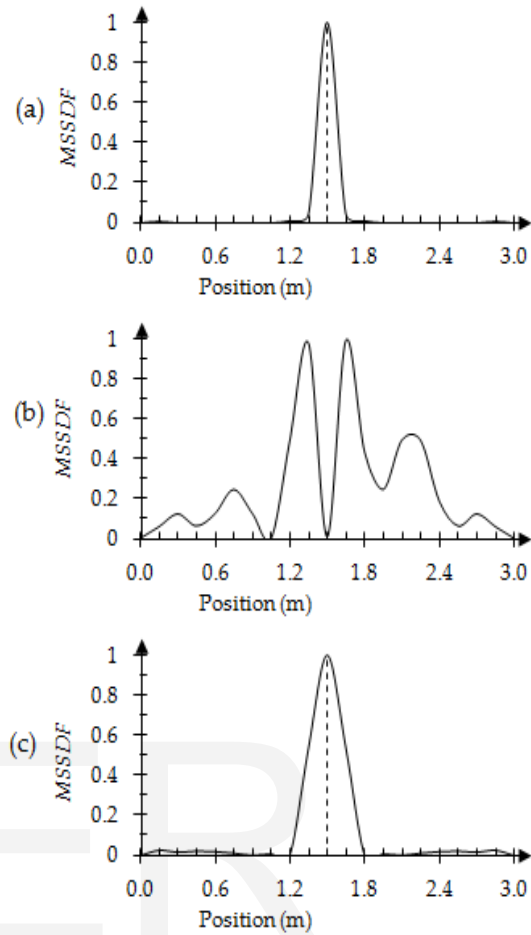


Fig. 4. Plots of damage parameter MSSDF vs. considered cross section position along the beam corresponding to the damage case 5 for: (a) the first mode shape, (b) the second mode shape, (c) the third mode shape

location method CDF given by (5). The latter is calculated considering the first three mode shapes. For better comparison of the results of the two damage indices, the CDF is normalized and is given as:

$$nCdf_{(k)} = \frac{\sum_{i=1}^N \left| \left( \Phi_{i,k}^{damaged} \right)'' - \left( \Phi_{i,k}^{undamaged} \right)'' \right|}{\max \left( \sum_{i=1}^N \left| \left( \Phi_{i,k=1,\dots,n}^{damaged} \right)'' - \left( \Phi_{i,k=1,\dots,n}^{undamaged} \right)'' \right| \right)} \quad (10)$$

As can be seen in Fig. 5, the two indices localize the damage for the case of single crack with .01 m depth and 0.1 m width. The peak of the plot is more pronounced for MSSDF indices, than for nCDF indices. The only exception is case 1 (Fig. 5a), when the crack location is near the end of the beam.

Fig. 6 shows the graphs of the damage indices for different crack depths and widths at the same location. As can be seen in Fig. 5d (case 4), Fig. 6a (case 6) and Fig. 6b (case 7) the crack depth reduction from 0.01 m to 0.005 m doesn't change significantly the plots of damage indices. At crack depth 0.002 m (Fig. 6b) the nCDF chart has many peak values and the MSSDF chart has a single peak, which is wider than those at crack depth 0.005 m (Fig. 6a) and crack depth 0.01 m (Fig. 5d). At crack width

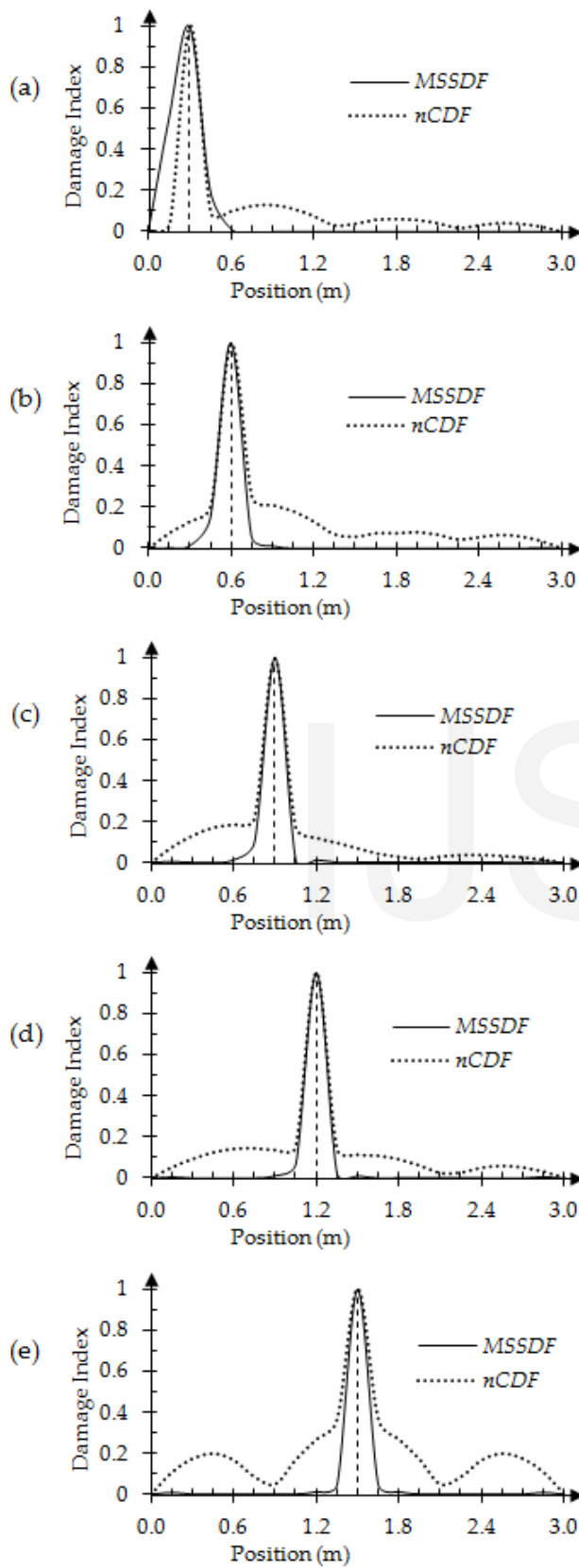


Fig. 5. Damage identification charts for cases of identical single damage at different locations along the beam: (a) case 1, (b) case 2, (c) case 3, (d) case 4, (e) case 5

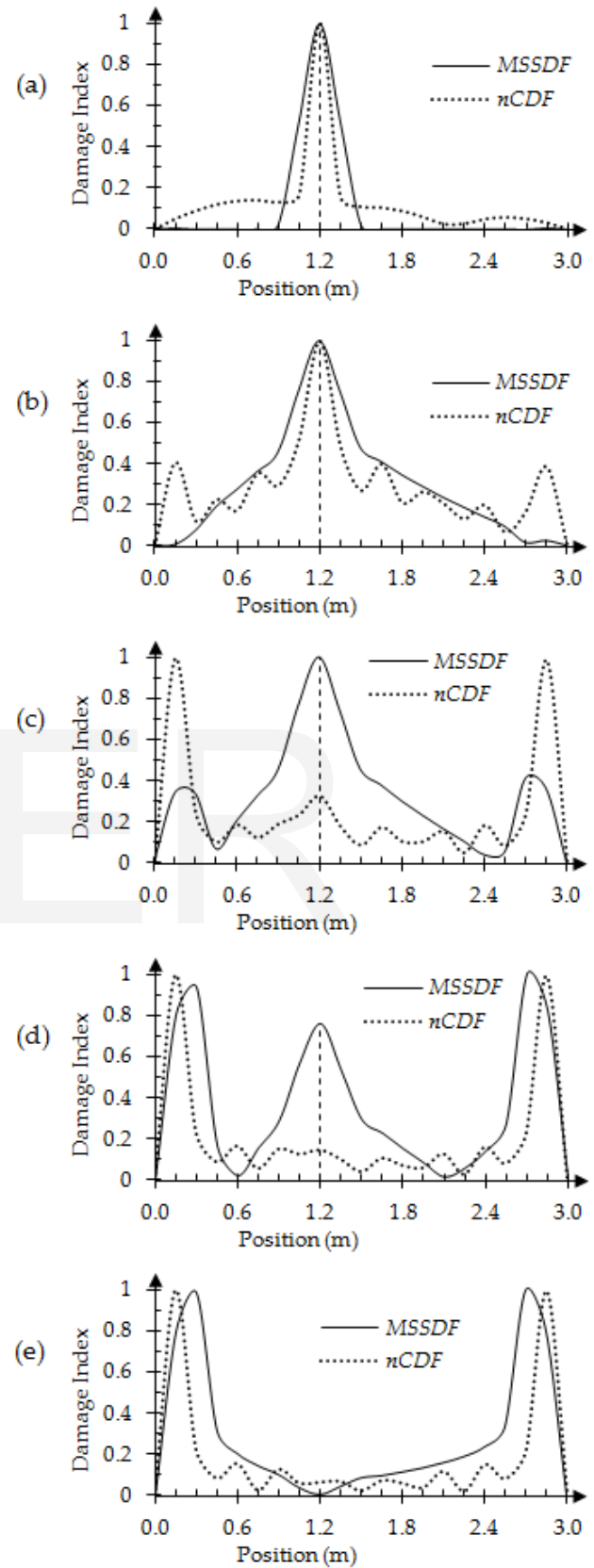


Fig. 6. Damage identification charts for cases of single damage located at 0.6 m from the left end of the beam: (a) case 6, (b) case 7, (c) case 8, (d) case 9, (e) case 10



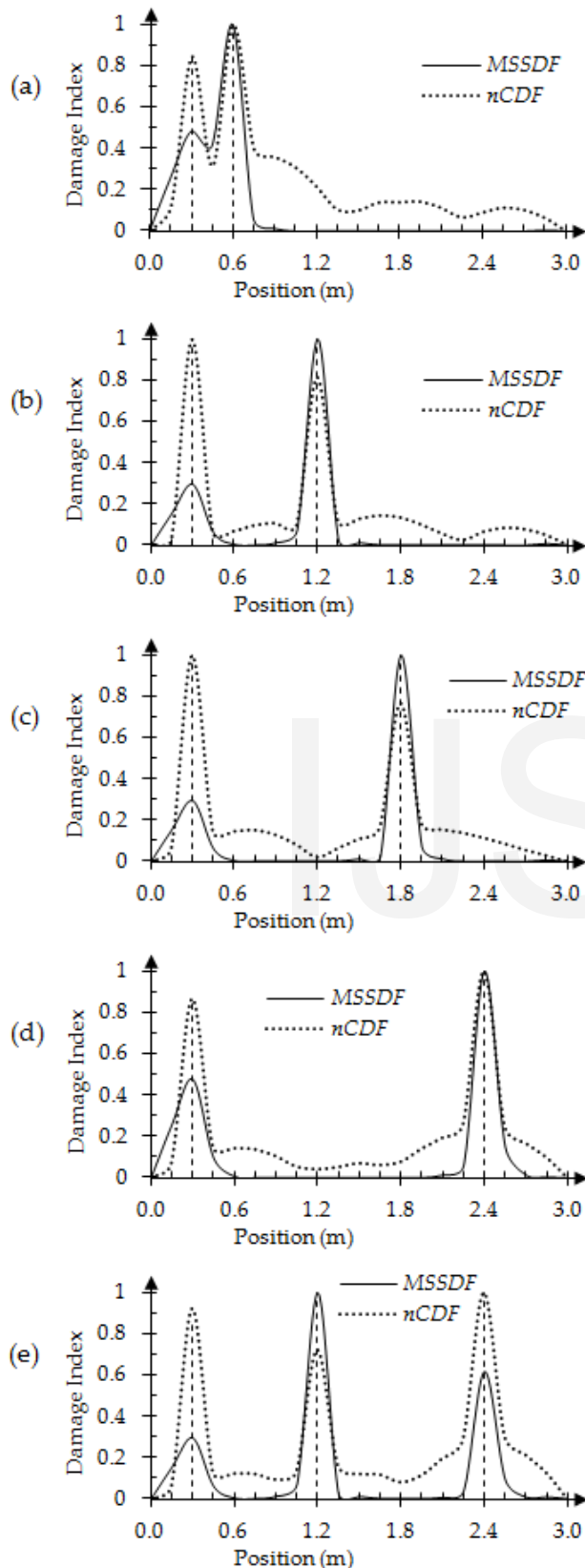


Fig. 7. Damage identification for multiple crack cases: (a) case 11, (b) case 12, (c) case 13, (d) case 14, (e) case 15

0.075 m (Fig. 6c) or less (Fig. 6d and Fig. 6e) the nCDF index isn't capable to locate the crack. The chart has extreme values near the ends of the beam, but there is no peak at crack location. At crack width 0.075 m (Fig. 6c) the MSSDF chart has three peaks and the maximum value is at the crack location. At crack width 0.05 m (Fig. 6d) the MSSDF chart has peak but it is smaller than the other two peaks, which are near the end of the beam. As seen in Fig. 6e, the MSSDF index doesn't localize the damage at crack width 0.025 m. Fig. 6 reveals that the damage detection index MSSDF is more sensitive to damage than nCDF.

Fig. 7 displays damage localization for multiple crack cases. The cracks parameters are: depth 0.01 m and width 0.1 m. The two damage indices correctly identify the damage locations. The CDF plot is characterized with many peaks, but at crack location the values are great than 0.9. The MSSDF plot has peaks only at damage location. Although all the cracks have equal parameters the MSSDF indices at damage location have different values as they vary from 0.29 to 1.

## 5. CONCLUSION

In this paper, a new damage location index based on the change in the mode shape slope is proposed. The damage index MSSDF is investigated for the first three mode shapes at two crack locations. Fifteen numerical finite element models of damaged beams are used to assess the validity and limitation of the method. The results obtained by method based on MSSDF indices are compared with those of other well-known damage location method in the literature, CDF. Both methods identify the location of the single or multiple damages with a precision for crack with depth more than 2% of height of the cross section and width equal to the width of the cross section. The MSSDF index is more sensitive to damage than CDF. For multiple damage cases the MSSDF indicator chart has peak at each damage location as the value of the peak doesn't correspond to damage parameters. Unlike the CDF indices, which use the data from three and more vibration modes, the MSSDF indicators consider the first mode shape. The numerical results demonstrate that MSSDF indicator is efficient tool for locating single or multiple damages in a simply supported beam.

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